Influence of Symmetry Energy Density on Isovector Giant Dipole Resonance

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Introduction

The centroid energies (E_{cen}) and electric polarizabilities (α_D) of nuclei undergoing modes of Giant Resonance (GR) can be extracted from experiment and compared with theoretical predictions to investigate Nuclear Matter (NM) properties and their effects on modes of GR.

Previous research using 33 Skyrme interactions established constraints on 3 NM properties but found no correlation between the E_{cen} of the Isovector Giant Dipole Resonance (IVGDR) and the symmetry energy coefficient (J) and its first, second, and third derivatives (L, K_{sym}, and Q_{sym} respectively) [1]. The goal of this project is to narrow down the 33 interactions based on the established constraints and determine whether a correlation with J emerges.

Objectives

1. Narrow down the Skyrme interactions that satisfy established constraints on the incompressibility coeficcient (K_{NM} = 210-240 MeV), effective mass (m^*/m = 0.7-0.9), and enhancement coefficent (κ = 0.25-0.70) of the energy weighted sum rule of the IVGDR.
2. Calculate predictions of E_{cen} and α_D of the IVGDR for each interaction.
3. Determine whether correlations exist between the calculated results and each of the relevant NM properties (J, L, K_{sym}, Q_{sym}).
4. Constrain any NM properties associated with strong correlations using experimental values of E_{cen} and α_D.

Calculations of E_{cen} and α_D are carried out for ^{40,48}Ca, ^{90}Zr, ^{116}Sn, and ^{208}Pb nuclei.

Model

The nucleus is treated as a many-body quantum system with interactions due to the strong force denoted by V_{ij} . The hamiltonian is as follows:

$$H = \sum_i \frac{p_i^2}{2m} + \sum_{i<j} V_{ij} \, .$$

The large number of terms in $H|\psi\rangle = E|\psi\rangle$ makes solving for the ground state not feasible. To simplify the problem, each particle is treated as subject to an average field with V_{ij} taking the form of different Skyrme interaction parametrizations [1].

Hartree-Fock

The overall wavefunction of the nucleus must be antisymmetric, as it is comprised of fermions. Here, ψ_i are the single particle wavefunctions.

$$\Psi(x_1, x_2, ...x_N) = \frac{1}{\sqrt{N}} \begin{vmatrix} \psi_1(x_1) & \cdots & \psi_1(x_N) \\ \vdots & \ddots & \vdots \\ \psi_N(x_1) & \cdots & \psi_N(x_N) \end{vmatrix} .$$

The use of a Slater determinant enforces antisymmetry. The ground state of the wavefunction is determined by finding the single particle wavefunctions that minimize the ground state energy, $E = \langle \Psi | H | \Psi \rangle$, through the variational process ($\psi_i \rightarrow \psi_i + \delta\psi_i$), obtaining the Hartree-Fock equation.

Hartree-Fock Continued

$$-\frac{\hbar}{2m}\Delta\psi_i(\mathbf{r}) + U_H(\mathbf{r})\psi_i(\mathbf{r}) - \int U_F(\mathbf{r},\mathbf{r}')\psi_i(\mathbf{r}')d^3\mathbf{r}' = e_i\psi_i(\mathbf{r})$$

Here, $U_H(\mathbf{r})$ and $U_F(\mathbf{r})$ are the direct and exchange potentials given in terms of the two-body interaction and the single particle wave functions. The Hartree-Fock equations are solved iteratively using the Woods-Saxon potential as an initial guess.

Random Phase Approximation

RPA is used to arrive at the strength function $S(E)$ (where $|j\rangle$ are the RPA states with energies E_j), using the scattering operator F_{LM}. The moments of the strength function are calculated and used to determine E_{cen} and α_D.

$$S(E) = \sum_j |\langle 0 | F_{LM} | j \rangle|^2 \, \delta(E_j - E_0)$$

$$F_{LM} = \frac{Z}{A} \sum_n f(r_n) Y_{LM}(n) - \frac{N}{A} \sum_p f(r_p) Y_{LM}(p)$$

$$m_k = \int_{E_1}^{E_2} E^k S(E) dE \, , \quad E_{cen} = \frac{m_1}{m_0} \, , \quad \alpha_D = \frac{24\pi e^2}{9} m_{-1}$$

NM Properties and Correlations

The NM properties we are concerned with are defined by the following equations, where ρ_0 is the saturation density, E_0 is the ground state energy, and E_{sym} is the symmetry energy.

$$K_{NM} = 9\rho_0^2 \frac{\partial^2 E_0}{\partial \rho^2}\Big|_{\rho_0} \, , \quad J = E_{sym}[\rho_0] \, , \quad L = 3\rho_0 \frac{\partial E_{sym}}{\partial \rho}\Big|_{\rho_0}$$

$$K_{sym} = 9\rho_0^2 \frac{\partial^2 E_{sym}}{\partial \rho^2}\Big|_{\rho_0} \, , \quad Q_{sym} = 27\rho_0^3 \frac{\partial^3 E_{sym}}{\partial \rho^3}\Big|_{\rho_0}$$

Correlations between NM properties and E_{cen} and α_D are calculated using the Pearson linear correlation coefficient, defined by the following equation:

$$r_{X,Y} = \frac{cov(X,Y)}{\sigma_X \sigma_Y} = \frac{\sum_i (x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum_i (x_i - \bar{x})^2} \sqrt{\sum_i (y_i - \bar{y})^2}}$$

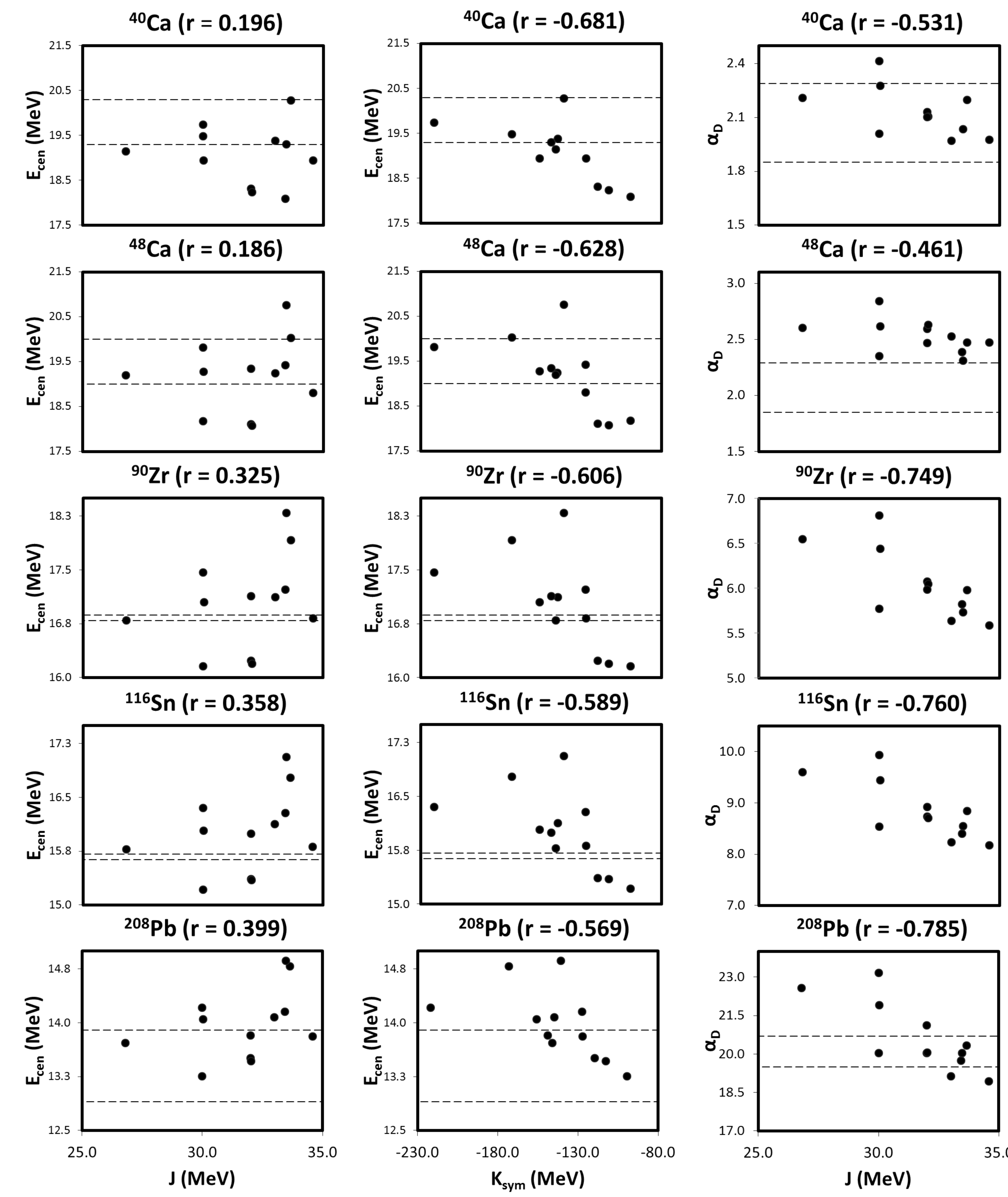
Linear correlation coefficients are calculated for each of the 5 nuclei, with the average correlations shown below. The correlations seen with the set of interactions used in prior research are shown for comparison.

Mean Correlations Between NM properties and E_{cen} and α_D of the IVGDR

	K_{NM}	J	L	K_{sym}	Q_{sym}	m^*/m	κ
33 Interactions (E_{cen})	0.00	-0.34	-0.40	-0.30	0.41	-0.60	0.84
12 Interactions (E_{cen})	-0.32	0.29	0.17	-0.62	-0.23	0.32	0.64
33 Interactions (α_D)	-0.14	0.27	0.52	0.43	-0.65	0.20	-0.14
12 Interactions (α_D)	-0.14	-0.66	-0.09	0.31	-0.50	0.07	0.30

The following figures visually demonstrate the relationship between NM properties and E_{cen} and α_D of the IVGDR for several representative cases. The first column shows E_{cen} against J, the second shows E_{cen} against K_{sym}, and the third shows α_D against J. The dashed lines indicate the experimental ranges [2]-[7]. No evidence of significant correlation between E_{cen} and J is demonstrated, while low to medium correlations are seen in the other two columns.

Graphical Representation



Conclusion

Removing the interactions that did not satisfy the established NM constraints did not reveal a correlation between E_{cen} and J, though K_{sym} does show a low correlation. The correlations with α_D demonstrated a low to medium correlation with J and a low correlation with Q_{sym}, neither of which are significant enough to justify imposing constraints based on experimental data. While these correlations are indicative of some influence of J on α_D, the results using E_{cen} demonstrate the same lack of influence of J on the IVGDR as was seen in past research conducted without the NM contstraints imposed here.

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